Human capital and the size distribution of firms*

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Abstract

Countries with a lower fraction of workers with secondary education have smaller firms. We set up a model of occupational choice where individuals have primary, secondary or tertiary education. A more educated work force raises firm size and productivity. More educated workers earn higher wages, and hence among educated individuals only the more able become entrepreneurs. We find that within the framework of our model, different educational attainments can explain one third of the difference in average firm size between the US and Mexico. While improved educational attainments hence imply an increase in firm size over time, a fall in the price of capital together with capital-skill complementarity acts in the opposite direction, something that can explain a relatively constant average firm size in the US since the late 1970's. Our policy experiments highlight how public employment and the skill bias in public hiring additionally affect firm size and productivity.

JEL Classification: J24, J45, E24, H30, O11.

Keywords: firm size, educational attainment, entrepreneurship, college premium, high school premium, public employment.

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1 Introduction

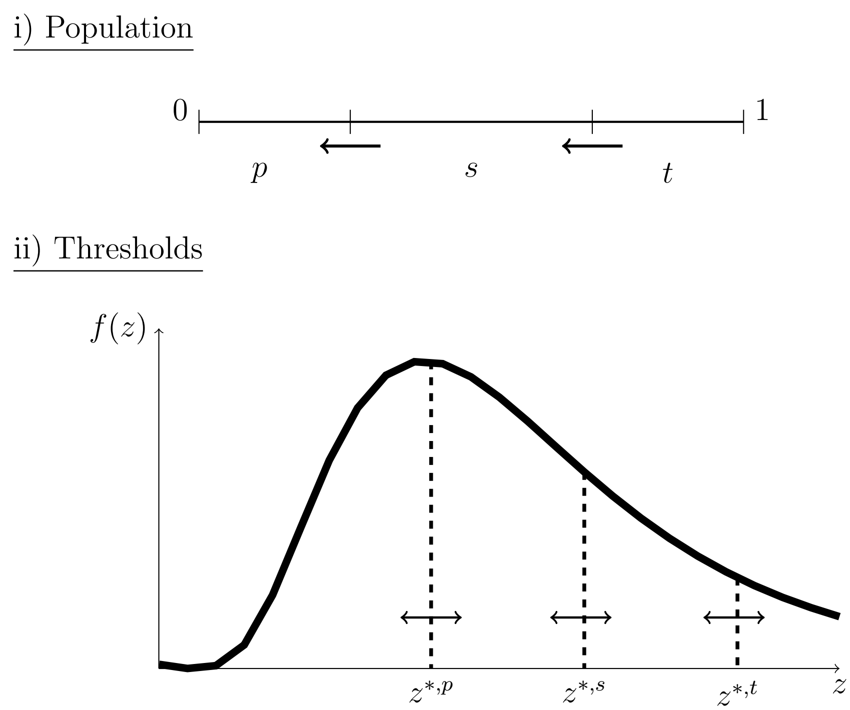
The size and productivity of firms differ widely across countries. In developing countries, the business landscape is characterized by small, informal, and less productive firms, and few large firms. Across Europe, small firms are more frequent in southern countries. Differences in firm size and productivity have direct implications for aggregate total factor productivity, output, and growth.[1] On the other hand, at least since Lucas [1988], the idea that human capital and education are important for economic growth has received a lot of attention. However, the joint effects of differences in human capital on firm size distributions and productivity and growth have received less attention.

We first investigate the link between education and firm size using data from two leading surveys: the Enterprise survey and the Global Entrepreneurship Monitor survey. We find that average firm size is positively related to the percentage of workers with secondary education rather than the share of college educated. This result is robust to the inclusion of a variety of different controls regarding financial development or institutional quality. We then propose a model where fewer educated individuals lead to a landscape of small and unproductive firms. In particular, our model proposes a novel micro-channel by which education affects productivity – the occupational choice of low educated individuals and their managerial talent.

Figure 1 displays the intuition behind our model's mechanism. Abstracting from the arrows, the upper graph of the figure displays how a population is composed of primary (p), secondary (s), and tertiary (t) educated individuals. The lower graph displays a probability density distribution for the managerial talent (z) of the population. Three thresholds indicate the managerial talent of the first individual to become an entrepreneur with primary $z^{*,p}$, secondary $z^{*,s}$ and tertiary $z^{*,t}$ education respectively. Let us first consider a world in which the distribution of managerial talent is the same for individuals of all three education groups. If individuals with more education earn higher wages they face higher opportunity costs of setting up their own firm, and hence the threshold for becoming an entrepreneur is higher for tertiary educated individuals compared to secondary or primary educated. Everything else equal, a country with a larger share of primary educated individuals will have a larger number of entrepreneurs with lower managerial ability and hence more small firms.

[1] See IDB [2010] or Lora *et al.* [2001] for evidence on firm size distributions in Latin America, and Davis and Henrekson [1999] or Kumar *et al.* [2001] for firm size distributions in Europe. Braguinsky *et al.* [2011] find that the shrinking size of firms in Portugal is linked to the country's reduced aggregate productivity.

Figure 1: Increase in educational attainment and occupational choices



An increase in average educational attainment due to fewer primary and more secondary and/or tertiary educated individuals, as depicted by the arrows in the upper graph, will have two effects. On the one hand, there are more individuals who have a high threshold for becoming an entrepreneur which increases average managerial talent. On the other hand, as wages and profits change, managerial thresholds for all education groups may shift. Arrows pointing to the left and right of the original thresholds indicate that they can move either way, depending on the initial situation as well as on the magnitude of the change in educational attainment. This second effect can potentially strengthen or weaken the first effect. Furthermore, one can envision that workers with more education draw their managerial ability from a better distribution which would amplify the positive effect of education on firm size.

We set up a model to quantify these effects. Agents in our model economy are endowed with an education level (primary, secondary, or tertiary) and a managerial ability. More educated individuals draw their managerial ability from a better distribution. According to their education and managerial ability, agents decide whether to become entrepreneurs or workers. Production requires capital, unskilled labor, and skilled labor, aggregated in a CES production function, similar to Krusell *et al.* [2000]. Considering three education groups, we depart from the standard dichotomy of unskilled and skilled labor in the production function. To reconcile the two approaches we assume the following setting. Workers with primary education only carry out unskilled jobs. Workers with tertiary education perform skilled jobs. Workers with secondary education can carry out unskilled jobs better than workers with primary education but they also perform skilled jobs, albeit not as well as college graduates.

We calibrate our model to match US current data and the time series of US college and high-school premia. We follow the approach of existing literature and use Latin America as a case study, comparing the United States and Mexico. Within our model, differences in educational endowments between the two countries explain about 37% and 36% of the differences in average firm size and GDP per capita respectively. Occupational choices, and managerial talent that depends on an individual's education amplify the effect of differences in education on productivity and output. The GDP per capita difference in our model is 6 percent larger compared to the one generated in a model with a single representative firm. Using educational attainment data for over 100 countries, our model is able to replicate the empirical relationship between average firm size and educational attainment and, in particular, the stronger relationship with the share of secondary educated individuals.

Our model also has time-series implications. Since the late 1970's, average firm size in the United States has stayed fairly constant, despite a strong increase in educational attainment. The model is able to replicate this fact. The positive effect of education on firm size is counteracted by the fall in the price of capital.

We then use our model to highlight the additional effects on firm size and productivity that arise from a skill-bias in public employment. In our model educational attainment of the population is exogenous. However, the number of educated workers and entrepreneurs in the private sector is determined by how many individuals, and in particular skilled individuals the public sector hires. Even for a given educational attainment of the population, firm size and productivity can thus differ under different public employment scenarios. Our findings regarding the effects of skill-biased public employment are in line with the discussion in Hamermesh [1996] regarding its negative effects for economic development. We find that firms' input choices are altered towards less capital and skilled labor and towards more unskilled workers. Moreover, public employment and its bias towards skilled workers actually raises firm size. Because by hiring more workers, the public sector pushes wages up, affecting occupational choices of the marginal entrepreneurs who before ran smaller firms and now chose paid employment.

Our paper mainly contributes to the literature on firm size distributions. Different from the current paper, this literature tends to focus on frictions that cause deviations from efficient firm size distributions. Cabral and Mata [2003] and Erosa [2001] argue that financial frictions restrain the growth of firms.[2] Hsieh and Klenow [2009] consider how distortions affecting marginal products of labor and capital lead to a departure from an efficient firm size distribution. Other explanations rely on policy aspects (e.g., Guner *et al.* [2008]) or institutions (e.g., Grobovšec [2013]). Antunes and Cavalcanti [2007] and Amaral and Quintin [2006] highlight the role of informality for smaller average firms in Latin America. Empirical studies suggest that these explanations are all relevant, with limited access to credit, labor market regulations, corruption, and entry costs having a positive influence on informality, and a negative influence on firm size (see Loayza [1997], Chong and Gradstein [2007], or Johnson *et al.* [1998]).

We also contribute to the literature on public sector employment where most theoretical papers proposes models with homogeneous workers and analyze crowding-out effects for private sector employment and effects on wages; see e.g. Finn [1998] in an RBC model or Gomes [2015] in a search and matching model. Albrecht *et al.* [2016] and Gomes [2016] who both analyze how workers with different qualifications select themselves into the public and private sector are among the few papers that consider potentially distinct effects across skill groups. Prior to that Domeij and Ljungqvist [2006] pointed out that the expansion of the Swedish public sector, that hired more low-skilled workers, was able to explain part of the difference in the evolution of the skill premium between the United States and Sweden. Regarding outcomes, to the best of our knowledge Cavalcanti and Rodrigues dos Santos [2014] is the only other paper that analyses how public employment affects occupational choices and entrepreneurship. The authors propose a model where well paid public sector jobs distort occupational choices of individuals as well as firms' input choices, leading to sizable output losses. In line with our results, the authors find that lower levels of public employment can increase entrepreneurship.

Finally, there is a vast literature that studies the relationship between human capital and aggregate growth and productivity; from classical works of Barro [1991] and Mankiw *et al.* [1992] to more recent papers by Gennaioli *et al.* [2013] and Erosa *et al.* [2010].

Closest to the current paper are the works by Roys and Seshadri [2014] and Poschke [2014]

[2]See the survey by Buera *et al.* [2015] for additional references.

that also consider how human capital affects both firm size distributions and productivity and growth. The former paper focuses on how occupational choice and human capital accumulation decisions amplify the effect of education on firm size, productivity, and development. Different from the current paper, the authors abstain from modeling capital-skill complementarity and educational wage premia but focus on wage premia paid by larger firms instead. Poschke [2014] presents exhaustive empirical evidence on the link between firm size distributions and economic development, and proposes a model that is able to replicate the empirical relationships. In his model most able entrepreneurs are in a better position to exploit technological advances, and hence with development fewer individuals become entrepreneurs, and they run larger firms.

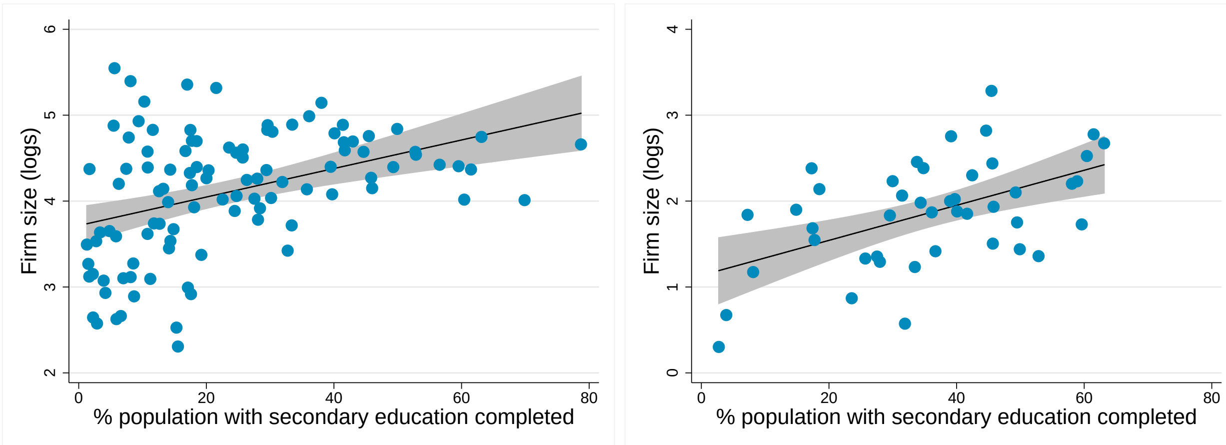
To the best of our knowledge ours is the first paper to empirically establish the positive and significant correlation of intermediate levels of education with average firm size and to propose a model that is able to generate this relationship. The remainder of this paper is organized as follows. Section 2 presents the empirical evidence on the relationship between the educational level of a country's workforce and average firm size. Section 3 describes the model and Section 4 the calibration strategy. Then, Section 5 presents and discusses the results. In Section 6 we conduct our policy experiments focusing on the skill bias in public employment, after which Section 7 concludes.

2 Firm size distribution and educational attainment

We investigate the relationship between education and firm size using data from two surveys: the World Bank Enterprise Survey and the Global Entrepreneurship Monitor survey. The Enterprise Survey has gathered data on 130,000 non-agricultural firms in 135 emerging markets and developing economies since 2002. The survey only samples firms with five or more employees. It collects information on the characteristics of the firm (at the plant-level) and its business conditions. For a detailed description of the survey and its advantages see García-Santana and Ramos [2015]. We construct the average firm size for each country, using the variable "Average Number of Permanent, Full Time Workers".

We also use data from the Global Entrepreneurship Monitor (GEM) survey. This survey is conducted by the London Business School and Babson College, and it covers more than 50 countries. The GEM survey captures mainly small and informal firms. Hence, by focusing on the left tail of the firm size distribution, it complements well the Enterprise survey. A

Figure 2: Firm size and fraction of secondary educated population



(a) Enterprise survey (b) Global Entrepreneurship Survey

more detailed description of this survey can be found in Poschke [2014]; for a comparison with other surveys see García-Santana and Ramos [2015]. From the GEM survey we use the variable "average firm size," calculated by Poschke [2014] for the period 2001–2005.

As our main exercise we run a cross-country regression of the log of average firm size on educational attainment. We consider two measures of educational attainment: the share of individuals with completed secondary education and some college, and the share of individuals with completed tertiary education (in %). Our data for educational attainment are from Barro and Lee [2013]. In our baseline regression we include as controls the log of the population and the log of GDP per capita taken from the Penn World Tables. The former has been suggested as a proxy for the size of a country's domestic market (see Kumar *et al.* [2001]). GDP per capita on the other hand captures many other elements that potentially correlate with firm size like financial development or the quality of institutions. Nevertheless, we carry out extensive robustness checks where we include indicators for institutional quality and variables related to financial development into our baseline regression. We also run our regression without controlling for population size. These and other additional results as well as summary statistics for all variables are displayed in Appendix I.

Figure 2 shows the unconditional correlation between average firm size (in logs) and the fraction of the population with secondary education. We observe a strong positive correlation in both surveys. Table 1 shows the results from our main regression. Column (1) presents results when we only include the fraction of secondary and tertiary educated individuals. In both surveys, only the coefficient on the former is significant at the 1 percent level.

Table 1: Average employment per firm and educational attainment

	Enterprise survey				GEM survey			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Secondary education	0.015***	0.009**		0.011**	0.019***	0.012**		0.014**
	(3.12)	(2.14)		(2.28)	(3.24)	(2.29)		(2.46)
Tertiary education	0.006		0.003	-0.013	0.006		-0.004	-0.013
	(0.41)		(0.25)	(-0.85)	(0.45)		(-0.35)	(-0.97)
Income per capita		0.201***	0.282***	0.219***		0.323**	0.494***	0.395***
		(2.96)	(4.20)	(3.08)		(2.58)	(3.33)	(2.71)
Population		0.136***	0.130***	0.142***		-0.375***	-0.537***	-0.438***
		(3.50)	(3.25)	(3.59)		(-3.17)	(-3.92)	(-3.24)
Observations	104	100	100	100	44	44	44	44
R-squared	0.164	0.316	0.284	0.321	0.275	0.427	0.353	0.440

*Notes: Data on educational attainment of the population over the age of 25 are from the Barro-Lee dataset for 2005. Tertiary education refers to the fraction of the population that has a college degree. Secondary education refers to the fraction of the population that has completed high school but does not hold a college degree. Data on population and income per capita are from the Penn World Tables. For the Enterprise survey, we compute the log of a country's average firm size. For the Global Entrepreneurship Monitor survey, we use the log of a country's average firm employment, as calculated by Poschke [2014]. The t-statistics are shown in brackets.*** indicates significance at the 1% level, ** indicates significance at 5% level, and * indicates significance at the 10% level.*

In columns (2) and (3) we add the log of GDP per capita and population and, in turn, each measure of educational attainment. Finally, column (4) presents results for the most complete specification that includes all controls.

The R-squared measures indicate that differences in educational attainment can account for 16-28% of the variation in average firm size across countries. In particular, average firm size turns out to be positively related to the fraction of the population that has completed secondary education, but not to the fraction of college graduates. The coefficient for the share of secondary educated is statistically significant at the 5 percent level and has a similar magnitude in both data sets – around 0.01. This implies that in a country where 65% of the population have secondary education (e.g. Germany), average firm size is 27% larger compared to a country with only 38% secondary educated (e.g. Italy). We also run a regression of the dispersion in firm size (calculated using standard errors) on educational attainment; see Table A12 in Appendix I. We find a positive correlation between the dispersion in firm size and the fraction of secondary educated workers. This is in line with Poschke [2014] who finds dispersion in firm size to increase with economic development.

As mentioned before, we confirm the robustness of our results along many different dimensions. Two results are worth highlighting. When using a different sample that includes OECD countries only, we estimate a positive and significant coefficient for the share of tertiary educated individuals but only when also controlling for secondary education. Hence, for richer countries tertiary education also matters for firm size, but never independently of secondary education. Second, when using average years of schooling and its square, instead of the share of secondary and tertiary educated, we estimate a negative and significant coefficient for years squared which suggests a decreasing relationship between additional years of education and average firm size. However, this last result is less robust because it is only significant in one of our samples.

All our results hence point to the fact that average firm size is closely related to the pool of workers with an intermediate level of education. A model for the relationship between average firm size and human capital thus needs to differentiate between at least three levels of education. Moreover, certifications like high-school diplomas or college degrees introduce discontinuities into how markets reward labor services, something that is reflected in college and high school wage premia and that a model with a continuous measure of education would not be able to capture.

3 Model

We build a model with three levels of education. In our model economy à la Lucas [1978] there is a single representative household and a government. The household is made up of a continuum of members with different levels of education e, with $e = p, s, t$. A fraction, p, of individuals has primary education, a fraction, s, has completed secondary education, and a fraction, t, has completed tertiary education, with $p + s + t = 1$. According to their educational level and managerial abilities, household members become either workers or entrepreneurs. Entrepreneurs produce a homogeneous good by using unskilled labor, skilled labor, capital, and their ability as inputs. A household decides about levels of consumption and savings, given the joint income of all household members.

Household The household maximizes the infinite sum of discounted utilities given by

$$\sum_{t=0}^{\infty} \beta^t log(C_t), \tag{1}$$

where C_t denotes total household consumption at time t and $\beta \in (0, 1)$ is the discount factor. The household's total size is normalized to unity. Since we focus on steady states and for expositional clarity, we omit the time subscript, t, from the description of the model.

Endowments Each household member has one unit of productive time that he/she supplies inelastically. Household members differ in their level of education e and managerial ability, z_i, distributed in $Z = [0, \overline{z}]$, and with cdf $F_e(z_i)$ and density $f_e(z_i)$ depending on an individual's level of education. The household assigns occupations to its members depending on their abilities and education. They can become either workers or entrepreneurs. Managerial ability of the marginal entrepreneur of primary, secondary or tertiary education is denoted by $z^{*,p}$, $z^{*,s}$, and $z^{*,t}$ respectively.

Production Each entrepreneur, i, has access to the same technology, rents capital, and hires different types of workers. Workers with primary education are hired as unskilled workers, $n_{p,i}$, and workers with a university degree are hired as skilled labor, $h_{t,i}$. Finally, workers with a high school diploma can be hired for unskilled work, $n_{s,i}$, or for skilled work, $h_{s,i}$. Firms produce a single good according to the following CES production function

$$y_i = f(n_{p,i}, n_{s,i}, h_{s,i}, h_{t,i}, k_i) = A z_i^{(1-\gamma)} [\mu (X_i^n)^\sigma + (1-\mu)[\lambda (q k_i)^\rho + (1-\lambda)(X_i^h)^\rho]^{\frac{\sigma}{\rho}}]^{\frac{\gamma}{\sigma}}, \quad (2)$$

where ρ and σ govern the elasticities of substitution between inputs. Total factor productivity (TFP) is denoted by A, and q denotes the relative productivity of capital goods (or the inverse of their relative price). The production function differs from Krusell *et al.* [2000] in two main aspects. First, $\gamma \in (0, 1)$ is the span-of-control parameter. The scale of production is increasing in the enhanced span-of-control (i.e., the entrepreneur's ability, z_i). Second, the unskilled and skilled labor inputs, X_i^n and X_i^h, respectively, are aggregations of workers with different education levels, given by:

$$X_i^n = ((\epsilon^n n_{p,i})^{\psi^n} + (n_{s,i})^{\psi^n})^{\frac{1}{\psi^n}} \quad (3)$$

$$X_i^h = ((\epsilon^h h_{s,i})^{\psi^h} + (h_{t,i})^{\psi^h})^{\frac{1}{\psi^h}}, \quad (4)$$

where $1/(1-\psi^n)$ and $1/(1-\psi^h)$ are the elasticities of substitution between workers with different education levels within unskilled and skilled labor respectively. If $\psi^n = 1$ ($\psi^h = 1$) then primary (secondary) and secondary (tertiary) educated workers are perfect substitutes for carrying out unskilled (skilled) jobs. The parameter ϵ^n indicates the productivity discount for workers with primary education relative to those with a high school degree when carrying out unskilled jobs. Similarly, ϵ^h reflects the productivity discount for workers with a high school degree relative to university graduates performing skilled tasks.

Entrepreneurs Entrepreneurs choose the number of workers, their education, and their position as unskilled or skilled workers, and capital to maximize their firms' profits. Given the production function, they always choose a strictly positive amount of all inputs. Given the wages per education level (w^p, w^s, w^t) and a rental rate for capital (r^k) the entrepreneur's problem is given by

$$\max_{\{n_{p,i},n_{s,i},h_{s,i},h_{t,i},k_i\}} \pi(z_i) = y_i - w^p n_{p,i} - w^s n_{s,i} - w^s h_{s,i} - w^t h_{t,i} - r k_i. \tag{5}$$

The first-order conditions are:

$$f_{n_p} = w^p \tag{6}$$

$$f_{n_s} = f'_{h_s} = w^s \tag{7}$$

$$f_{h_t} = w^t \tag{8}$$

$$f_k = r. \tag{9}$$

The entrepreneur equates the marginal productivity of each factor to its cost. Combining the first-order conditions for labor, we can show that:

$$\frac{h_{s,i}}{h_{t,i}} = \left(\frac{w^t (\epsilon^h)^{\psi^h}}{w^s} \right)^{\frac{1}{1-\psi^h}}, \tag{10}$$

$$\frac{n_{p,i}}{n_{s,i}} = \left(\frac{w^s (\epsilon^n)^{\psi^n}}{w^p} \right)^{\frac{1}{1-\psi^n}}. \tag{11}$$

Firms hire more workers with a high school degree for skilled positions if their productivity discount is small, and if the college premium is high. For unskilled positions, firms hires more workers with secondary education if the productivity discount for workers with only primary education is high, and if the high school wage premium is low.

Notice that firms' profits do not depend directly on the education of the manager, but only on his/her managerial ability. A second aspect worth noticing is that all firms hire the same skill mix, independently of their scale. The implication of this is that, in the model, changes in educational attainment of the population only affect average firm size if the share of entrepreneurs in the economy is altered.

The Household's problem The household chooses a level of consumption and savings, and the optimal occupation for each household member, $\{C, K', z^{*,p}, z^{*,s}, z^{*,t}\}$, in order to maximize the infinite sum of discounted utilities (Equation 1) subject to

$$\begin{aligned} C + K' &= rK(1-\tau) + (1-\delta)K + (1-\tau)[pw^p F_p(z^{*,p}) + sw^s F_s(z^{*,s}) + tw^t F_t(z^{*,t}) + \\ &\quad + t\int_{z^{*,t}}^{\overline{z}} \pi(z_i, .) f_t(z)dz + s\int_{z^{*,s}}^{\overline{z}} \pi(z_i, .) f_s(z)dz + p\int_{z^{*,p}}^{\overline{z}} \pi(z_i, .) f_p(z)dz], \end{aligned}$$

where π represents a firm's before-tax profits. The household income includes the capital income, the wage income, and the profits of its members who are entrepreneurs. All income is taxed at rate τ. The solution to the household's problem is characterized by the following first-order conditions, evaluated at the steady state:

$$r = \frac{1}{(1-\tau)}(\frac{1}{\beta} - 1 + \delta), \tag{12}$$

$$w^p = \pi(z^{*,p}, .), \tag{13}$$

$$w^s = \pi(z^{*,s}, .). \tag{14}$$

$$w^t = \pi(z^{*,t}, .). \tag{15}$$

Condition (12) is the standard Euler equation for optimal capital accumulation, which determines the equilibrium interest rate. Conditions (13)–(15) are similar to Lucas' [1978] condition for the "marginal' entrepreneur. Wage payments have to equal the profits individuals expect to make as entrepreneurs. Household members with primary, secondary, and tertiary education, and managerial abilities $z^{*,p}$, $z^{*,s}$ and $z^{*,t}$, respectively, are indifferent between working or setting up a firm.

Government The government in this economy hires workers to produce the government consumption good. It collects taxes on wages, profits, and capital income. The tax rate is determined to balance the budget, given by

$$\begin{aligned} &l^g[tw^t + sw^s + pw^p] = \tau[rK + w^p F_p(z^{*,p})p + w^s F_s(z^{*,s})s + w^t F_t(z^{*,t})t + \\ &p\int_{z^{*,p}}^{\overline{z}} \pi(z_i, .) f_p(z)dz + s\int_{z^{*,s}}^{\overline{z}} \pi(z_i, ;) f_s(z)dz + t\int_{z^{*,t}}^{\overline{z}} \pi(z_i, ;) f_t(z)dz]. \end{aligned} \tag{16}$$

The government chooses the level of public sector employment, l^g, and hires workers of each education level. Contrary to what we know from the data we assume that the public sector hires the same proportion from each education group. We make this assumption to be able to better isolate the model's main mechanism. We relax it when conducting policy experiments in Section 6. Wages paid by the government are equal to those paid in the private sector, a common assumption in models with frictionless labor markets.[3] Following the convention in national accounts, government consumption in the model is measured at factor costs and is given by the public sector wage bill $g = l^g[tw^t + sw^s + pw^p]$.

Equilibrium In equilibrium, all five markets must clear: the three labor markets plus the capital and goods markets. Denote the demand for primary, secondary, tertiary educated labor services, and capital by an entrepreneur with ability z_i by $n_{p,i}(z_i, w^p, w^s, w^t, r)$, $n_{s,i}(z_i, w^p, w^s, w^t, r)$, $h_{s,i}(z_i, w^p, w^s, w^t, r)$, $h_{t,i}(z_i, w^p, w^s, w^t, r)$, and $k_i(z_i, w^p, w^s, w^t, r)$, respectively. For workers with primary education, the labor market clears when:

$$\begin{aligned} P \equiv F_p(z^{*,p})p &= pl^g + p\int_{z^{*,p}}^{\overline{z}} n_{p,i}(z_i, w^p, w^s, w^t, r)f_p(z)dz + s\int_{z^{*,s}}^{\overline{z}} n_{p,i}(z_i, w^p, w^s, w^t, r)f_s(z)dz \\ &+t\int_{z^{*,t}}^{\overline{z}} n_{p,i}(z_i, w^p, w^s, w^t, r)f_t(z)dz. \end{aligned} \tag{17}$$

The aggregate supply of workers with primary education, P, must equal the sum of labor demands by all entrepreneurs and the government. For workers with a secondary and tertiary education, the labor markets clear when:

$$\begin{aligned} S \equiv F_s(z^{*,s})s &= sl^g + p\int_{z^{*,p}}^{\overline{z}} n_{s,i}(z_i, w^p, w^s, w^t, r)f_p(z)dz + p\int_{z^{*,p}}^{\overline{z}} h_{s,i}(z_i, w^p, w^s, w^t, r)f_p(z)dz \\ &+s\int_{z^{*,s}}^{\overline{z}} n_{s,i}(z_i, w^p, w^s, w^t, r)f_s(z)dz + s\int_{z^{*,s}}^{\overline{z}} h_{s,i}(z_i, w^p, w^s, w^t, r)f_s(z)dz + \\ &t\int_{z^{*,t}}^{\overline{z}} n_{s,i}(z_i, w^p, w^s, w^t, r)f_t(z)dz + t\int_{z^{*,t}}^{\overline{z}} h_{s,i}(z_i, w^p, w^s, w^t, r)f_t(z)dz \end{aligned} \tag{18}$$

[3]Empirical evidence indicates that wages in the public sector are higher than in the private sector, particularly for low-educated workers; see Gregory and Borland [1999]. In our model, given that the government always hires an exogenous number of workers, a higher public sector wage would only affect the government's wage bill but not its ability to hire; see Gomes [2016] for a discussion on the effects of different public sector wage premia in a model with search and matching frictions. Cavalcanti and Rodrigues dos Santos [2014] study in detail the role of public sector wages on individuals' occupational choices.

and

$$
\begin{aligned}
T \equiv F_t(z^{*,t})t &= tl^g + p\int_{z^{*,p}}^{\overline{z}} h_{t,i}(z_i, w^p, w^s, w^t, r) f_p(z)dz + s\int_{z^{*,s}}^{\overline{z}} h_{t,i}(z_i, w^p, w^s, w^t, r) f_s(z)dz \\
&\quad + t\int_{z^{*,t}}^{\overline{z}} h_{t,i}(z_i, w^p, w^s, w^t, r) f_t(z)dz. \qquad (19)
\end{aligned}
$$

The market clearing condition for capital is given by:

$$
\begin{aligned}
K &= p\int_{z^{*,p}}^{\overline{z}} k_i(z_i, w^p, w^s, w^t, r) f_p(z)dz + s\int_{z^{*,s}}^{\overline{z}} k_i(z_i, w^p, w^s, w^t, r) f_s(z)dz \\
&\quad + t\int_{z^{*,t}}^{\overline{z}} k_i(z_i, w^p, w^s, w^t, r) f_t(z)dz. \qquad (20)
\end{aligned}
$$

With $y_i(z_i, w^p, w^s, w^t, r)$ being the supply of goods by any entrepreneur of ability z_i, for market clearing in the goods market, we require

$$
\begin{aligned}
p\int_{z^{*,p}}^{\overline{z}} y_i(z_i, w^p, w^s, w^t, r) f^p(z)dz + s\int_{z^{*,s}}^{\overline{z}} y_i(z_i, w^p, w^s, w^t, r) f^s(z)dz \\
+ t\int_{z^{*,t}}^{\overline{z}} y_i(z_i, w^p, w^s, w^t, r) f^t(z)dz = C + \delta K. \qquad (21)
\end{aligned}
$$

To calculate GDP, we add the government value added to private sector output such that $GDP = C + \delta K + g$. We can now define a competitive equilibrium for the model economy in steady state. Given a government policy $\{l^g\}$ and a sequence of prices for labor and capital $\{w^p, w^s, w^t, r\}$, a competitive equilibrium is a collection of thresholds $\{z^{*,p} z^{*,s}, z^{*,t}\}$, a tax rate $\{\tau\}$, and allocations $\{P, S, T, K', C\}$, such that:

1. $\{z^{*,p} z^{*,s}, z^{*,t}\}$ solve the household's problem (i.e., equations (13)–(15) hold);
2. the rental rate is determined by the Euler equation (12);
3. the five markets for goods, capital, primary, secondary, and tertiary educated labor clear (i.e., equations (17)–(21) hold);
4. the tax rate, τ, balances the government's budget (i.e., equation (16) holds).

4 Calibration

Table 2 displays our chosen parameter values. We first set certain parameters exogenously, based on available evidence. We fix the discount factor β to 0.96 and the depreciation rate, δ, to 8%. Kydland and Prescott [1982] choose β to match a steady state interest rate of 4%. In equilibrium this rate plus the depreciation rate equals the marginal productivity of capital, and hence given the capital share, the authors target the capital-output ratio. Similarly, in our model both parameters together with the equilibrium tax rate determine the before-tax gross steady state interest rate (see Equation 12). Firms choose the optimal amount of capital such as to equate this interest rate to the marginal productivity of capital (see Equation 9). Given parameters for the production function, and in particular λ, both parameter values also determine the private capital-output ratio in our model.[4]

According to the OECD [2011], in 2008, public employment made up 14.6% of the US labor force. In our benchmark case, we consider skill-neutral public employment, in the sense that the government hires the same proportion of primary, secondary, and tertiary educated as there are in the population. According to data from Barro and Lee [2013] in 2005, the educational composition of the US population age 25 and above was as follows: 31.1%, 58.1%, and 10.9% had tertiary, secondary, and primary education respectively. For q, the parameter for the relative productivity of capital goods, we use the inverse of the price of investment relative to the price of consumption, taken from the FRED database. This ratio is normalized to 1 for 2005.

The remaining parameters related to the distribution of abilities and the production function are calibrated to minimize the distance between the model's statistics and targets from US data. Our calibration strategy proceeds in two steps. First we calibrate a slightly modified economy with only one unique distribution of managerial ability for all three education groups and thus fewer parameters, see Table A13 in Appendix II. We then use the so-found parameters as starting values for the calibration of our full model with three different distributions for managerial ability. Even though in a general equilibrium model all parameters affect all targets, we briefly discuss the data moments that each parameter is most likely to determine.

We take as the empirical counterpart of a unit of production, establishments rather than firms. We assume that managerial ability for entrepreneurs of each education group is dis-

[4] One can show that $f_k = \frac{y}{k}\Omega$ where $\Omega = \gamma\lambda(qk_i)^\rho(\frac{Az_i^{1-\gamma}}{y})^{\sigma/\gamma}[(\frac{y}{Az_i^{1-\gamma}})^{\sigma/\gamma} - \mu(X_i^n)^\rho]^{1-\rho/\sigma}$.

Table 2: Baseline calibration

Parameters set exogenously	**Value**	**Source**
Discount factor (β)	0.960	Kydland and Prescott (1982)
Depreciation rate (δ)	0.080	Kydland and Prescott (1982)
Public employment (l^g)	0.146	OECD
2000's		
Fraction of primary educated (p^{00})	0.109	Barro and Lee (2013)
Fraction of secondary educated (s^{00})	0.581	Barro and Lee (2013)
Fraction of tertiary educated (t^{00})	0.310	Barro and Lee (2013)
Inverse price of capital 2000's (q^{00})	1	Normalization
TFP 2000's (A_{00})	1	Normalization
1990's		
Fraction of primary educated 90's (p^{90})	0.201	Barro and Lee (2013)
Fraction of secondary educated 90's (s^{90})	0.533	Barro and Lee (2013)
Fraction of tertiary educated 90's(t^{90})	0.266	Barro and Lee (2013)
Inverse price of capital 90's (q^{90})	0.760	FRED Database
1970's		
Fraction of primary educated 70's (p^{70})	0.379	Barro and Lee (2013)
Fraction of secondary educated 70's (s^{70})	0.497	Barro and Lee (2013)
Fraction of tertiary educated 70's(t^{70})	0.124	Barro and Lee (2013)
Inverse price of capital 70's (q^{70})	0.510	FRED Database
Calibrated parameters	**Value**	**Target**
Distribution of ability		
Shape parameter, tertiary (α_t)	1.156	Mean establishment size,
Shape parameter, secondary (α_s)	1.007	– entrepreneur secondary educated
Shape parameter, primary (α_p)	0.967	– entrepreneur primary educated
Scale parameter (ξ)	0.112	Establishment share (< 10)
Location parameter (θ)	2.027	Establishment share (> 100)
		Employment share (> 100)
Production function		
Span-of-Control (γ)	0.857	Profit share
Weights		
Unskilled labor in production (μ)	0.309	Labor share
Capital in Production (λ)	0.549	Capital-output ratio
Secondary educ. workers in skilled input (ϵ^h)	0.613	College premium (2000)
Primary educ. workers in unskilled input (ϵ^n)	0.756	High school premium (2000)
Substitutability		
Within skilled jobs (ψ^h)	0.661	College and High school premia 90's & 70's
Within unskilled jobs (ψ^n)	0.667	College and High school premia 90's & 70's
Between capital and skilled labor (σ)	0.858	College and High school premia 90's & 70's
Between capital and unskilled labour (ρ)	-1.120	College and High school premia 90's & 70's
Time series		
TFP 90s (A_{90})	0.958	GDP per capita 90's
TFP 70s (A_{70})	0.873	GDP per capita 70's

tributed according to a generalized Pareto distribution – $f_e(z) = \frac{1}{\xi}(1 + \alpha_e \frac{z-\theta}{\xi})^{-(1/\alpha_e+1)}$ – with scale parameter ξ and location parameter θ. For each education group the distribution is characterized by a different shape parameter α_e with $e = t, s, p$. According to the Business Dynamic Statistics of the US Census, mean establishment size was 17.5. The shape parameter α_t for the distribution of tertiary educated is set to 1.16 to match this number. The Survey of Business Owners (SBO 2007) has information about firm size and the education of managers. We restrict our sample to firms with managers who are majority owners. The average size of establishments with primary and secondary educated entrepreneurs was equal to 50% and 83% of mean size. We set shape parameters α_p and α_s to 0.97 and 1.01 to target these numbers. According to the Business Dynamic Statistics of the US Census, establishments with more than 100 employees make up 2.6% of all establishments. These establishments account for 44.5% of all employment. Small establishments with fewer than 10 workers make up 70.4% of all establishments. Parameters, ξ and θ are calibrated to 0.11 and 2.03 respectively. Given that we have five parameters and six targets, we assign weight 5/6 to each of these targets.

We now turn to the parameters of the production function. The weight of capital in production, λ, is set to 0.55 to target a private capital-output ratio of 2, as established for the United States in Kamps [2006]. According to Goldin and Katz [2009], the college premium in the 2000 Census was 61% and the high school premium was 31%. To match these numbers, the relative productivities of secondary to tertiary (ϵ^h) and primary to secondary (ϵ^n) workers are calibrated to 0.61 and 0.76 respectively. Between 2005 and the second quarter of 2007, the average wage compensation made up around 63% of GDP, and corporate profits together with proprietors' income were equal to 13.3% of GDP (Bureau of Economic Analysis). These targets determine the weight of unskilled labor in production, μ, of 0.31 and the span-of-control parameter, γ, of 0.86.

As our production function differs from those typically used in literature, we cannot match the elasticities of substitution to existing empirical evidence. To calibrate parameters, ρ, σ, ψ^n, and ψ^h governing the elasticities of substitution between skilled and unskilled labor and between the three different levels of education, we introduce a time series dimension. We target the evolution of the US college and high school premia at two additional points in time: 1970's and 1990's. Between the 1970's and the 2000's, the share of tertiary and secondary educated individuals increased considerably as did wage premia for both groups. Similar to Krusell *et al.* [2000] we use the fall in the relative price of capital together with skill-capital complementarity to replicate this feature in the data.

We thus simulate our economy for three periods (2000's, 90's, 70's), adjusting the relative price of capital, and the educational composition of the population for each decade. We also estimate relative TFP in the 1990's and the 1970's to target GDP per capita in these decades relative to the 2000's.

The quantitative predictions of our model depend very much on the elasticities of substitution between the three inputs to production. Parameters ρ and σ, as well as ψ^n and ψ^h determine these elasticities. We set parameters ψ^n and ψ^h to 0.67 and 0.66 respectively. Parameters ρ and σ are set to -1.12 and 0.86. Our estimated values for ψ^n and ψ^h imply elasticities of substitution of around 3 for secondary and primary educated in unskilled jobs and secondary and tertiary educated in skilled jobs. Our parameter values for σ and ρ imply an elasticity of substitution between skilled and unskilled labor of 7 and 0.5 for the elasticity of substitution between capital and skilled labor.

Our parameter values for σ and ρ are different from values in literature, in particular from those in Krusell *et al.* [2000]. While our value for σ is quite similar to the one estimated by Polgreen and Silos [2008] using NIPA data, our value for ρ is much smaller. This is due to the fact that different from models typically used for estimating values of σ and ρ, our model includes three educational groups. Due to the fact that in our model workers with secondary education can carry out both, skilled and unskilled jobs, our production function already has some "built-in" substitutability. This implies that as the price of capital and thus low-skilled wages fall, in our model secondary educated workers can switch from doing unskilled jobs to carrying out skilled jobs. This channel reduces the pressure on demand for skilled labor and the rise in skill premia is weakened. Hence, compared to a model with only two types of skills, our model requires more skill complementarity to be able to match the observed rise in skill premia.[5]

Table 3 displays the calibration targets next to the model's statistics, as well as some additional moments that were not targeted. Our model matches the data well, including the non-targeted moments. We somewhat overestimate the average size of firms run by tertiary educated individuals but underestimate the fraction of large firms run by them. Once we target average firm size, the self-employment rate in our model is determined. Targeting an average establishment size of 17.46, together with a share of public employment of 14.6% fixes the entrepreneurship rate in our model at 5% ($\frac{1-0.146-e}{e} = 17.46$; $e = 0.05$). The model thus underestimates the share of self-employed in the population of 7% as reported by the

[5] In Tables A18 to A20 in Appendix II, we present an alternative calibration with a very high elasticity of substitution within skilled and unskilled jobs ($\psi^n = \psi^h = 0.976$). Such a calibration is unable to capture the increase in high-school and college premia over time.

Table 3: Calibration targets and model values

Targeted moments	**Source**	**Data**	**Model**
Mean establishment	US Census	17.46	17.57
Relative size establishment primary educated manager	SBO(2007)	0.50	0.50
Relative size establishment secondary educated manager	SBO(2007)	0.83	0.83
Establishment share, < 10 employees	US Census	0.70	0.73
Establishment share, > 100 employees	US Census	0.03	0.03
Employment share, > 100 employees	US Census	0.45	0.45
Capital-output-ratio	Kamps (2006)	2.00	1.99
Profits to GDP	BEA	0.13	0.13
Wage bill	BEA	0.63	0.63
College Premium 2000	Goldin and Katz 2009	0.61	0.61
College Premium 1990	Goldin and Katz 2009	0.55	0.54
College Premium 1970	Goldin and Katz 2009	0.47	0.46
High School Premium 2000	Goldin and Katz 2009	0.31	0.31
High School Premium 1990	Goldin and Katz 2009	0.28	0.29
High School Premium 1970	Goldin and Katz 2009	0.23	0.23
GDP per capita 1990's relative to 2000's	BEA	0.82	0.82
GDP per capita 1970's relative to 2000's	BEA	0.55	0.53
Non-targeted moments	**Source**	**Data**	**Model**
Relative size establishment tertiary educated manager	SBO(2007)	1.27	1.90
Employment share, < 10 employees	US Census	0.15	0.19
Employment share, $20 - 99$ employees	US Census	0.30	0.25
Establishment share, $20 - 99$ employees	US Census	0.13	0.10
Fraction of large (> 100 employees) firms managed by			
tertiary educated	SBO(2007)	0.63	0.49
secondary educated	SBO(2007)	0.35	0.44
primary educated	SBO(2007)	0.02	0.06
Self-employment rate	OECD	0.07	0.05

OECD.[6]

5 Education and firm size

We evaluate the predictive power of our model in a cross-country setting and along the US time-series dimension. We start by adjusting the US current educational attainment to that of Mexico to understand how the educational composition of the labor force affects average firm size and productivity. We also replicate our empirical results from Section 2 by running the model for a large number of countries, adjusting the educational composition of their

[6]The OECD statistic is also similar to the fraction of unincorporated self-employed over total employment in the US of 6-7% as reported in Hipple [2010]. As the author points out, many data sources tend to count incorporated self-employed as employees, potentially also our source for average establishment size. In this case the most comparable rate to our model statistic is the fraction of unincorporated self-employed.

populations, and looking at the model's predictions for average firm size. Finally, we consider the time-series implications of our model for the United States.

5.1 United States versus Mexico

To evaluate the impact of the supply of skilled labor on firm size and productivity, we compare our benchmark economy to an identical economy that has the educational attainment of Mexico. In particular, we run our benchmark economy for the 2000's maintaining all parameters including the level of TFP from Table 3, only changing parameters t^{00}, s^{00} and p^{00}. According to data from Barro and Lee [2013] in 2005, the educational composition of the Mexican population age 25 and above was as follows: 9.1%, 16.0%, and 74.9% had tertiary, secondary, and primary education respectively.

Table 4 displays the benchmark results for the United States next to those for Mexico. In the last two columns we show the results from a representative firm model. The level of education of the Mexican labor force is much lower than that of the US labor force. But what does this imply for output and, in particular, for firm size and productivity?

In our model, the average firm size in Mexico is 13 - lower than in the US. In the data, the average firm in Mexico has 5 workers (Instituto Nacional de Estadística y Geografía) [2014]). Hence we can explain around 37% of the difference in average firm size. However, notice that this number masks the two effects depicted in Figure 1. Holding all entrepreneurial thresholds fixed at the US level, and hence assuming that in Mexico the same fraction of individuals from each education group as in the US chooses to set up a firm, we can construct a hypothetical measure for average firm size in Mexico. This is done by simply weighting those fractions by the shares of primary, secondary, and tertiary educated in Mexico. Under this assumption the average firm in Mexico would have 11 workers. However, profits and wages change, and thus thresholds in Mexico are different. Regarding wages, because skilled workers are much scarcer in Mexico, wages of secondary and tertiary educated are 36% and 21% higher, while wages of primary educated are 35.6% lower. On the other hand, and due to higher skilled wages, profits across all levels of managerial ability are 36.2% lower. We hence observe that all thresholds shift to the right, particularly for secondary and tertiary educated individuals. This is why, conditional on educational attainment of the entrepreneur, firms in Mexico are larger than in the US, particularly the average size of firms with a secondary or tertiary educated manager is twice as large. Nevertheless, average firm size in Mexico is

Table 4: Results: United States and Mexico

Statistic	Baseline Model		Representative firm	
	United States	Mexico	United States	Mexico
Firm size distribution				
Mean establishment size				
Overall	17.57	12.95	-	-
Entrepreneurs with primary education	8.74	10.35	-	-
Entrepreneurs with secondary education	14.57	40.17	-	-
Entrepreneurs with tertiary education	33.41	69.86	-	-
Establishment share, < 10 employees	0.73	0.80	-	-
Establishment share, > 100 employees	0.03	0.02	-	-
Employment share, < 10 employees	0.19	0.24	-	-
Employment share, $20 - 99$ employees	0.25	0.24	-	-
Employment share, > 100 employees	0.45	0.40	-	-
Fraction of large firms (> 100 employees) with				
Primary educated manager	0.06	0.63	-	-
Secondary educated manager	0.44	0.17	-	-
Tertiary educated manager	0.49	0.20	-	-
Dispersion in establishment size	108.0	100	-	-
Entrepreneurship				
Share of entrepreneurs	0.05	0.06	-	-
Fraction of entrepreneurs with				
Primary education	0.19	0.94	-	-
Secondary education	0.60	0.04	-	-
Tertiary education	0.22	0.02	-	-
Threshold to become entrepreneur				
Primary education	3.22	3.25	-	-
Secondary education	4.22	8.96	-	-
Tertiary education	6.81	12.93	-	-
Aggregate variables				
Private capital-output-ratio	1.99	1.06	1.99	1.02
Profits to GDP	0.13	0.13	0.13	0.13
Wagebill	0.63	0.75	0.63	0.75
College premium	0.61	0.44	0.62	0.43
High-school premium	0.31	1.77	0.32	1.84
College premium of entrepreneurs	1.29	0.74	-	-
High-school premium of entrepreneurs	0.67	2.88	-	-
Private sector output per worker	181.9	100	180.8	100
GDP per capita	181.7	100	177.2	100
Output per Establishment	246.9	100	-	-

Notes: For the representative firm model we use the parameter values for the production function from our baseline economy.

lower because there are many low ability entrepreneurs who only have primary education: 94% of all firms and 63% of firms with more than 100 workers are run by primary educated managers.

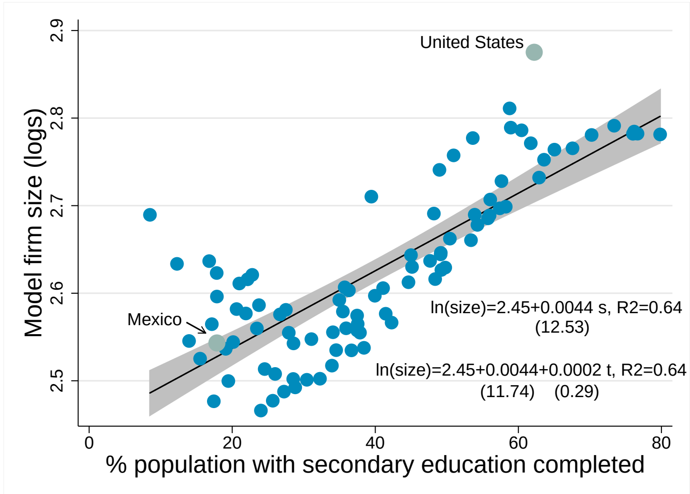
The different educational wage premia in Mexico drive differences in firms' input choices. Mexican workers with a secondary education earn 177% more than primary educated workers. The college wage premium is only 44% and thus lower than in the United States. These numbers are in line with empirical evidence. According to López-Acevedo [2001], workers with a college degree in Mexico earn 53% more than those with an upper secondary education, who in turn earn 70% more than those with primary education and 170% more than those with no schooling. Note that our measure of primary educated also includes individuals without schooling. As low-skilled labor is thus relatively cheap in Mexico firms substitute capital by primary educated workers, which leads to a lower capital-output ratio. In line with empirical findings for the US reviewed in van der Sluis, van Praag, and Vijverberg [2008], education premia in our model are higher for entrepreneurs' profits than for workers' wages. Furthermore, these differences are larger in the US compared to Mexico.

Productivity in Mexico is lower when comparing all measures. More individuals in Mexico set up firms than in the United States, and more entrepreneurs lower the average managerial talent. In addition, more entrepreneurs in Mexico are primary educated individuals which leads to even lower average managerial talent. This is why, private output per worker and GDP—the sum of private output and the government's wage bill—are also lower.

In the last part of Table 4 we display aggregate statistics from our model for both countries and compare them to those generated by a model with a representative firm. Similarly for the representative firm model, parameters for the US and Mexico are assumed to be the same, and we only adjust the distribution of education in each country. In our benchmark model, higher educational attainment in the United States implies that GDP per capita is 82% higher than in Mexico, compared to a difference of 77% generated by a representative firm model. Hence, taking into account the effect of education on the firm size distribution amplifies by 6% the effects of education on GDP. In the data, the ratio of GDP per capita US-Mexico is 3.23. Differences in educational attainment alone can hence account for 36% of the difference in GDP per capita.

In line with our empirical findings, dispersion in establishment size is higher in the US than in Mexico.

Figure 3: Mean firm size and educational attainment in model



Data: Educational Attainment: Barro-Lee dataset; Firm size: Model

5.2 Cross-country analysis

Using data on educational attainment from Barro and Lee [2013], we test how well the model is able to replicate the positive cross-country relationship between educational attainment and average firm size. We restrict our attention to countries that have at least 5 percent of the population in each of the three educational categories, and we run our benchmark model with all parameter values from Table 2, and we only change the share of primary, secondary and tertiary educated individuals for each country.[7]

Figure 3 shows the positive relationship between secondary educational attainment and the log of average firm size that arises from this exercise. The slope of the related regression is 0.0044, compared to 0.01 in our empirical analysis in Section 2. This confirms that also in our model, secondary education is closely related to average firm size. The R-squared is 0.63. Adding the fraction of those with tertiary education to the model's regression of the log of average firm size on secondary education marginally raises the R-squared to 0.64. This suggests that also in our model for a large sample of countries tertiary education only has a limited contribution for average firm size.

[7]A very small share of one educational group of workers implies that for this group marginal productivity and wages skyrocket. To account for these cases, we would have to generalize the model such that all workers could perform both skilled and unskilled jobs. In such a model, when there are very few secondary and tertiary educated workers, workers with primary education might take over skilled jobs. Similarly, when almost every worker has tertiary education some of them might perform unskilled jobs.

5.3 United States Time Series

The time series simulation of our model provides additional predictions that can be contrasted with data. Table 5 shows the evolution of average firm size in the US and the one predicted by the model. While during most of the 20th century average firm size in the US increased, looking at comparable data from the US Census's Business Dynamic Statistics available since the late 1970's, we do not observe any clear trend, with average establishment size fluctuating around 17.

Table 5: Determinants of the evolution of firm size in the US

Year	Data	Model		
		Baseline	Only changing education (P, S and T)	Only changing technology (q and A)
1970	-	**18.63**	13.91	**18.63**
1975	-	18.07	14.72	17.92
1980	17.00	17.47	15.38	17.31
1985	16.59	17.03	15.65	16.29
1990	17.11	16.83	15.87	15.48
1995	16.87	16.69	16.36	14.60
2000	18.11	16.88	16.82	13.88
2005	17.47	**17.57**	**17.57**	13.91
2010	16.79	17.66	17.66	13.91

Notes: Data on average establishment size are from the US Census' Business Dynamic Statistics available since 1977. For the baseline case, we vary the education shares in the population, the relative price of capital, and TFP estimated for three decades (2000's, 90's and 70's). In the last two columns we change in turn only the education shares and only the relative price of capital and TFP.

In our baseline model where we adjust education and technology over time, we also do not detect any clear trend in average firm size. However, this result is the consequence of two opposing forces that offset each other. The positive effect of education on firm size is counteracted by the fall in the price of capital. In the third column of Table 5 we display results when fixing technology at its 2005 values and only changing the educational attainment of the population. Increasing education from its 1970's level to today leads to an increase in firm size by 26%. On the other hand, fixing the education level in the 1970's and only improving technology decreases average firm size by 34%.[8] In the presence of capital-skill complementarity, a reduction in the price of capital raises wages of secondary and tertiary educated workers by more than profits, and hence fewer will become entrepreneurs. However, primary educated workers do not benefit as much, and given that profits increase, their entrepreneurial thresholds fall and many will become entrepreneurs. In the 1970's pri-

[8] See Table A14 in Appendix II for a similar table for the model with a unique talent distribution.

mary educated individuals made up almost 40% of the population, and hence with constant education shares the later effect dominates and average firm size falls (see last column).

6 The effects of public employment

The public sector is the largest employer in the economy, both in advanced and developing economies. Behar and Mok [2013] report that, on average for 194 countries, public sector employment accounts for 15% of total employment. Besides hiring a large fraction of the labor force, the public sector displays a clear bias towards skilled workers, equally common in both advanced and developing economies. Giordano *et al.* [2011] report that the average share of workers with tertiary education is 2.6 times higher in the public than in the private sector in Euro Area countries, ranging from 1.6 times in Belgium to 4.3 times in Portugal. In the case of Latin America, Mizala *et al.* [2010] report that the average years of education in the public sector are 3 to 6 years higher than in the private sector, while Panizza [2000] finds the public sector to hire on average 30% of workers with at least secondary education. Assaad [1998] describes an even larger skill bias in Egypt, where the government hired 70% of females and more than 40% of males with a high school diploma or a university degree.[9]

In our model where educational attainment of the population is exogenous, public employment determines the number of educated workers and entrepreneurs in the private sector. The framework of our model allows us to analyze both the impact of public employment as well as the effect of skill-biased public employment for entrepreneurship, firm productivity, and aggregate output. To guide the discussion, let us write the expression for average firm size in a country as

$$firm\ size = \frac{1 - \%\ public\ employment - \%\ entrepreneurs}{\%\ entrepreneurs}. \tag{22}$$

Increasing public employment has a direct negative effect on average firm size because given a fixed number of firms, there are fewer workers available in the private sector. However, the total effect on average firm size also depends on the effect on entrepreneurship.

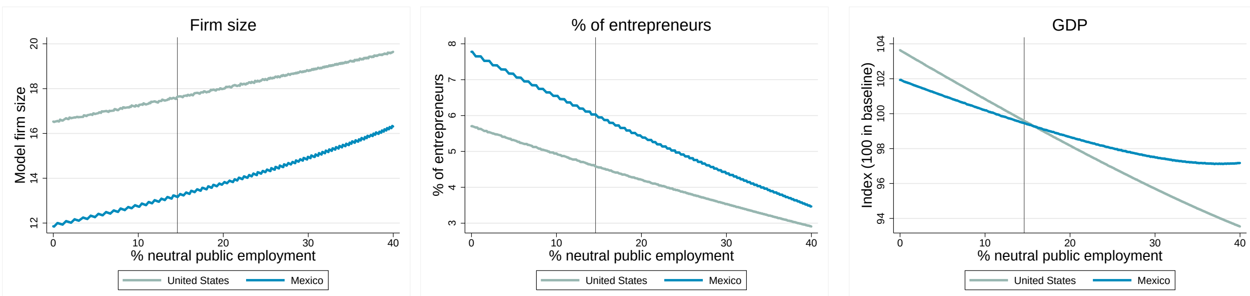
Figure 4 shows how average firm size, the fraction of entrepreneurs in the population, and GDP vary with the size of the public sector. Perhaps surprisingly, average firm size increases

[9] Note that contrary to our assumption, public employment in the US is also skill-biased. However, as we show in Table A17 in Appendix II, taking into account the actual educational composition of the US public sector has only minor effects on firm size.

with the level of public sector employment. By hiring more workers, the public sector pushes up all wages, affecting occupational choices of the marginal entrepreneurs who now chose paid employment.

Regarding GDP, public employment only has a small impact. A 15 percentage point increase in public employment reduces GDP by 2 and 4 percent in Mexico and the US respectively. GDP includes government consumption which is essentially the government's wage bill. As the public sector expands, wages increase and so does government consumption, independently of the productivity of public employees or the value of government services.

Figure 4: The effects of skill-neutral public employment

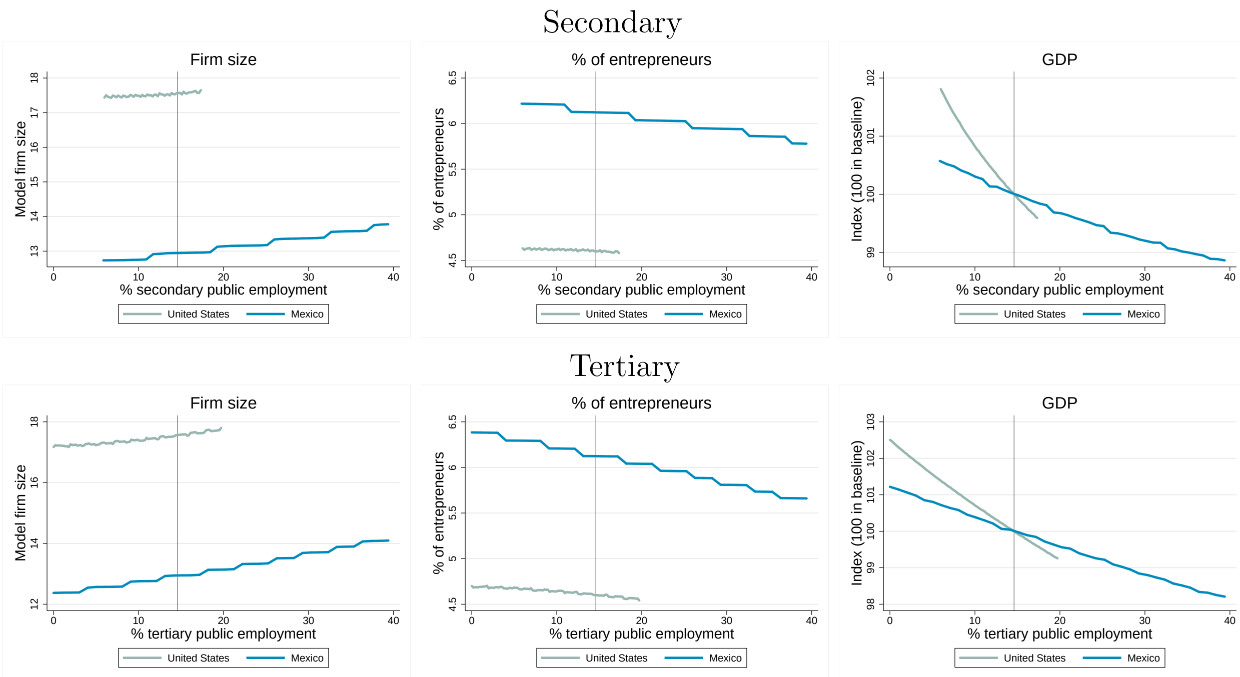


Note: vertical line indicates the baseline calibration.

We now turn to the effects of skill-biased public employment. In this exercise we keep the overall level of public employment at 14.6%, and we adjust the share of secondary and tertiary educated public employees, compensating with a change in the public hiring of primary educated workers. Figure 5 displays the results from this exercise. We find that increases in the government's wage bill via overall expansions of the public sector, or via hiring more skilled workers who earn higher wages have similar effects. Hiring more secondary or tertiary educated public employees raises firm size, reduces entrepreneurship, and has a small but negative effect on GDP. In line with Hamermesh [1996], skill-biased public employment alters firms' input choices towards less capital and skilled labor and towards more unskilled workers; see Figures A1 and A2 in Appendix II.

An expansion of the public sector has similar negative effects on GDP as a reduction in educational attainment. Within a representative firm model both scenarios lead to similar outcomes. However, within our model we are able to observe that effects on entrepreneurship and average firm size point into opposite directions. Whereas more public employment and skill-biased public employment lead to increases in average firm size, lower educational attainment in a country is associated with smaller firms. This is due to the fact that while

Figure 5: The effects of skill-biased public employment



Note: vertical line indicates the baseline calibration. Increases (decreases) in the share of secondary or tertiary educated public employees are obtained by reducing (increasing) the share of primary educated. The low shares of primary educated workers in the US and secondary and tertiary educated workers in Mexico place a lower and an upper bound on how many primary, secondary, and tertiary educated public sector workers can be hired.

public employment affects wages and hence decisions of marginal entrepreneurs, it has little effect on the overall managerial talent in an economy. Most entrepreneurs will not change their occupational choice as the public sector expands. While the public sector tends to be an absorber of skilled workers, contributing to the scarcity of skilled labor, it does not act as an absorber of managerial talent. On the other hand, a reduction in educational attainment lowers overall managerial talent.

7 Conclusion

Many alternative mechanisms have been proposed to explain why the business landscape in developing countries is characterized by small and less productive firms and few large firms. We find that, empirically, the educational attainment of a country's population is positively related to firm size, particularly the fraction of the population that has completed secondary education. This finding is robust to the inclusion of a variety of financial and institutional controls which are related to other explanations why firms in some countries are smaller.

In our model economy, a more educated labor force raises firm size and productivity. More educated workers earn higher wages, which leads to a more select group of entrepreneurs among educated individuals. Calibrated to the United States, we find that within our model differences in educational attainment alone have the potential to explain 37% and 36% of the differences in Mexican and US average firm size and GDP per capita, respectively.

Empirical findings have suggested an important role for secondary education in reducing income inequality (see Tilak [1989]). Our model proposes a micro mechanism of how a larger fraction of individuals with secondary education can lead to higher output, lower wage premia, and thus lower wage inequality. Regarding public hiring policies, we highlight how their effect on average firm size depends on the adjustment in entrepreneurship. In particular, we find that an increase in the public sector's wage bill – due to an overall expansion or a skill-bias in public employment – affects occupational choices of the marginal entrepreneurs who now chose paid employment. Our model thus allows us to measure the effects of public sector hiring that go beyond those already pointed out by previous studies, such as queuing or changes in skill premia.

Our model takes the education attainment of the population as given, but has interesting implications regarding incentives to accumulate human capital. For instance, we find that education premia are higher for entrepreneurs than for workers, and that this differences is larger in the US compared to Mexico. It would require a model with endogenous human capital formation to address how much differences in wage and entrepreneurial education premia can account for differences in educational attainment between the two countries. This suggests that endogeneizing individuals' human capital accumulation and in particular examining the role of public sector employment on education, could be an interesting road for future research.

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